

On the Theory of a New Maximum Observed in dc Transport in Modulated Quantum Hall Systems Near $\nu = 1/2$

Nataliya A. Zimbovskaya and Joseph L. Birman

Department of Physics, The City College of CUNY, New York, NY, 10031, USA

We propose a theory for the new maximum recently observed by Willett et al [Phys. Rev. Lett. **83**, 2624 (1999)] in the longitudinal magnetoresistance of a weakly modulated two-dimensional electron gas (2DEG) near filling factor $\nu = 1/2$ for the current driven along the modulation lines. The maximum is superimposed upon a new resonance structure. This occurs due to the geometric resonance of composite fermion cyclotron orbits with the period of modulation of the effective magnetic field. We propose here a semiquantitative theory of the dc magnetotransport in a modulated 2DEG near one half filling. Our analysis is based on the Boltzmann transport equation and it enables us to obtain this maximum which was neither observed nor predicted previously.

PACS numbers: 71.10 Pm, 73.40 Hm, 73.20 Dx

The present work is motivated by the new experimental results of dc transport experiments of Willett et al [1] in a two-dimensional electron gas (2DEG) in the fractional quantum Hall regime near half filling of the lowest Landau level ($\nu = 1/2$). The 2DEG was modulated with a small period density modulation applied in one direction. A minimum in the magnetic field dependence of the dc resistivity component for a current driven across the modulation lines, and a maximum in the magnetoresistance corresponding to a current driven along the modulation lines, was observed at about $\nu = 1/2$. Both minimum and maximum are superimposed upon a new resonance structure produced by the modulation in the region immediately adjacent to $\nu = 1/2$.

Similar structures corresponding to a special kind of geometric resonance (so called Weiss oscillations) were observed in dc transport experiments in both electrostatic and magnetic field modulated 2DEG in low magnetic fields [2, 3, 4, 5]. A theory of these oscillations in modulated 2DEG systems was first developed within the framework of a quantum mechanical approach [6, 7, 8]. An equivalent semiclassical approach to the analysis of these phenomena was first proposed by Beenakker [9] who pointed that Weiss oscillations could be explained by means of the guiding center drift of cyclotron orbits of the electrons in the presence of the modulating electric field. The most complete semiclassical consideration of magneto-transport in a modulated 2DEG is presented in Ref. [10] (See also Ref. [11]).

The minimum in the magnetoresistivity corresponding to a current driven across the modulation lines (ρ_{\perp}) at $\nu = 1/2$ was previously observed in the experiments [12]. A theoretical description of this feature of the magnetoresistivity ρ_{\perp} was presented in Refs. [13, 14, 15]. Here we focus on the maximum in the magnetic field dependence of the magnetoresistivity for a current driven along the modulation lines (ρ_{\parallel}). Such a maximum was never observed before and it was never predicted in theoretical studies. We develop a semiquantitative theory of the dc magnetotransport which gives a new explanation of this

maximum observed in the experiments [1].

Our work is based on the theory of the quantum Hall system at and near $\nu = 1/2$ proposed by Halperin, Lee and Read (HLR) [16, 17], which corresponds to the physical picture of the electrons decorated by attached quantum flux tubes. These are the relevant quasiparticles of the system – so called composite fermions (CF). The CFs are charged spinless fermionic quasiparticles which move in the reduced effective magnetic field $B_{eff} = B - 4\pi\hbar cn/e$ (n is the electron density) [23]. At $\nu = 1/2$ the CFs form a Fermi sea and exhibit a Fermi surface (FS). For the unmodulated 2DEG the CF-FS can be supposed to be a circle in quasimomenta space. Its radius p_F equals $\sqrt{4\pi n\hbar^2}$.

Density modulation influences the CF system through the direct effect of the modulating potential which can deform the CF-FS [22] and through the effect of an additional inhomogeneous magnetic field $\Delta B(\mathbf{r})$ proportional to the density modulation $\Delta n(\mathbf{r})$ ($\Delta B(\mathbf{r}) = -4\pi\hbar c\Delta n(\mathbf{r})/e$) [14]. To analyse the effect systematically we have to solve the Boltzmann transport equation for the CF distribution function in the presence of a spatially inhomogeneous disturbance due to the density modulation, similarly to Refs. [10, 15]. When, however, the CFs mean free path l is larger than the radius of their cyclotron orbit at the effective magnetic field R and the period of modulation λ we can obtain the desired response functions by means of simplified considerations based on the works of Beenakker [9] and Gerhardtts [11].

We start from the Lorentz force equations describing the CF motion along the orbit:

$$\frac{dp_x}{dt} = -\frac{e}{c}B(\mathbf{r})v_y; \quad \frac{dp_y}{dt} = \frac{e}{c}B(\mathbf{r})v_x, \quad (1)$$

where $p_{x,y}$ and $v_{x,y}$ are the components of the CF quasimomentum and velocity; $B(\mathbf{r}) = B_{eff} - 4\pi\hbar c\Delta n(\mathbf{r})/e$.

We will consider a single-harmonic sinusoidal density modulation of period $\lambda = 2\pi/g$ along the "y" direction: $\Delta n(\mathbf{r}) \equiv \Delta n(y) = \Delta n \sin(gy)$. We assume that the correction term $\Delta B(\mathbf{r})$ is small compared to B_{eff} .

Under this assumption we can write the CF velocity \mathbf{v} in the form $\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v}$, where \mathbf{v}_0 is the uniform-field velocity and the correction $\delta\mathbf{v}$ arises due to the inhomogeneity of the magnetic field. For a circular CF-FS we have: $v_{x0} = v_F \cos \Omega t$; $v_{y0} = v_F \sin \Omega t$; $\Delta n(y) \approx \Delta n \sin(gY - gR \cos \Omega t)$, where v_F is the CFs Fermi velocity, Ω is their cyclotron frequency in the effective magnetic field and Y is the "y" coordinate of the guiding center. Substituting these expressions for \mathbf{v} and $\Delta n(y)$ into Eq. (1) and keeping only first-order terms we obtain:

$$\frac{d(\delta v_x)}{dt} = -\Omega \delta v_y - \frac{\Delta B}{B_{eff}} \Omega v_F \sin \Omega t \sin(gY - gR \cos \Omega t);$$

$$\frac{d(\delta v_y)}{dt} = \Omega \delta v_x + \frac{\Delta B}{B_{eff}} \Omega v_F \cos \Omega t \sin(gY - gR \cos \Omega t). \quad (2)$$

We have to remark here that in the presence of the density modulation the CF's Fermi velocity gets a correction due to the modulation. To evaluate this correction we calculate the average of $\Delta n(y)$ over the cyclotron orbit. Expanding the functions $\cos(gR \cos \Omega t)$ and $\sin(gR \cos \Omega t)$ in Bessel functions we arrive at the following result for the averaged correction to the inhomogeneous density modulation:

$$\begin{aligned} \langle \Delta n(y) \rangle &\equiv \frac{\Delta n}{2\pi} \int_0^{2\pi} \sin(gY - gR \cos \psi) d\psi \\ &= \Delta n \sin(gY) J_0(gR). \end{aligned} \quad (3)$$

Here $\psi = \Omega t$.

The result (3) gives a spatially inhomogeneous correction to the chemical potential of the CFs and to their Fermi velocity:

$$\tilde{v}_F = v_F \sqrt{1 + \frac{\Delta n}{n} \sin(gY) J_0(gR)}.$$

The same holds for the scattering rate of CFs. In periodically modulated samples the scattering rate can be a function of a coordinates. Within the chosen geometry (the modulation is applied along the "y" direction) we can describe a periodic modulation of the relaxation time following Ref. [10]:

$$\frac{1}{\tau(y)} = \frac{1}{\tau} + r(y).$$

The correction $r(y)$ here is proportional to $\Delta n(y)/n$. However we can neglect these corrections to the CF Fermi velocity and their relaxation time in further calculations because the parameter $(\Delta n/n) = (\Delta B/B)$ is much smaller than the parameter $\Delta B/B_{eff}$. Thus omitted terms are in order of magnitude smaller than those which are kept. It is natural to suppose that to the first order in the modulating field the corrections δv_x and δv_y are periodic over the unperturbed cyclotron orbit. This

assumption is equivalent to that used in Ref. [11]. Under this assumption we can calculate averages of Eq. (2) over the cyclotron orbit. This gives us the following expressions for the components of the velocity of the guiding center drift V_x and V_y defined below:

$$\begin{aligned} V_x(Y) &= \langle \delta v_x \rangle = -\frac{v_F}{2\pi} \frac{\Delta B}{B_{eff}} \int_0^{2\pi} \cos \psi \sin(gY \\ &\quad - gR \cos \psi) d\psi = v_F \frac{\Delta B}{B_{eff}} \cos gY J_1(gR); \\ V_y(Y) &= \langle \delta v_y \rangle = -\frac{v_F}{2\pi} \frac{\Delta B}{B_{eff}} \int_0^{2\pi} \sin \psi \sin(gY \\ &\quad - gR \cos \psi) d\psi = 0. \end{aligned} \quad (4)$$

To evaluate the CF conductivity semiquantitatively we assume that the x component of the CF velocity can be written in the form $v_x(Y) = v_{x0} + V_x(Y)$. We also assume that the cyclotron frequency Ω can be replaced by the quantity $\Omega(Y) = \Omega + \langle \Delta \Omega(y) \rangle$ where $\langle \Delta \Omega(y) \rangle$ is the correction to the cyclotron frequency arising due to the inhomogeneity of the effective magnetic field averaged over the cyclotron orbit:

$$\begin{aligned} \Omega(Y) &= \Omega \left\{ 1 + \frac{\Delta B}{B_{eff}} \frac{1}{2\pi} \int_0^{2\pi} \sin(gY - gR \cos \psi) d\psi \right\} \\ &= \Omega \left\{ 1 + \frac{\Delta B}{B_{eff}} \sin(gY) J_0(gR) \right\}. \end{aligned} \quad (5)$$

We showed before [22] that in the semiquantitative analysis of the magnetotransport in modulated the 2DEG we can use the following approximation for the CF conductivity:

$$\sigma_{\alpha\beta}^{cf} \approx \frac{g}{2\pi} \int_{-\pi/g}^{\pi/g} \sigma_{\alpha\beta}^{cf}(Y) dY, \quad (6)$$

where

$$\sigma_{\alpha\beta}^{cf}(Y) = \frac{e^2 m_c \tau}{2\pi \hbar^2} \sum_k \frac{v_{k\beta}(Y) v_{-k\beta}(Y)}{1 + ik\Omega(Y)\tau}. \quad (7)$$

Here m_c is the CF cyclotron mass; τ is the relaxation time; $v_{k\beta}(Y)$ are the Fourier transforms for the CF velocity components:

$$\begin{aligned} v_{kx} &= \frac{v_F}{2} (\delta_{k,1} + \delta_{k,-1}) + V_x(Y) \delta_{k,0}; \\ v_{ky} &= \frac{iv_F}{2} (\delta_{k,1} - \delta_{k,-1}). \end{aligned}$$

Keeping terms of the order of $(\Delta B/B_{eff})^2$ or larger we obtain the following approximations for the CF conductivity components $\left(\frac{\Delta B}{B_{eff}} \Omega \tau = \frac{\Delta n}{n} \frac{p_F l}{\hbar} = \frac{\Delta n}{n} k_F l \right)$:

$$\sigma_{xx}^{cf} \approx \frac{\sigma_0}{1 + (\Omega\tau)^2} \left\{ 1 + \frac{3}{2} \left(\frac{\Delta n}{n} k_F l \right)^2 \frac{J_0^2(gR)}{1 + (\Omega\tau)^2} \right\}$$

$$+ \sigma_0 \left(\frac{\Delta B}{B_{eff}} \right)^2 J_1^2(gR); \quad (8)$$

$$\sigma_{yy}^{cf} \approx \frac{\sigma_0}{1 + (\Omega\tau)^2} \left\{ 1 + \frac{3}{2} \left(\frac{\Delta n}{n} k_F l \right)^2 \frac{J_0^2(gR)}{1 + (\Omega\tau)^2} \right\}; \quad (9)$$

$$\begin{aligned} \sigma_{xy}^{cf} &= -\sigma_{yx}^{cf} \\ &\approx \frac{\sigma_0 \Omega \tau}{1 + (\Omega\tau)^2} \left\{ 1 + \frac{1}{2} \left(\frac{\Delta n}{n} k_F l \right)^2 \frac{J_0^2(gR)}{1 + (\Omega\tau)^2} \right\}. \end{aligned} \quad (10)$$

where $\sigma_0 = ne^2 l / p_F$ is the CF conductivity in a homogeneous magnetic field.

The last term in the expression for σ_{xx}^{cf} describes the contribution from CFs diffusing along the "x" direction which arises due to the guiding center drift. To show it we can calculate the corresponding contribution to the diffusion coefficient δD . Following Refs. [9, 11] we write:

$$\delta D = \tau \frac{g}{2\pi} \int_{-\pi/g}^{\pi/g} V_x^2(Y) dY. \quad (11)$$

This term δD gives the additional contribution to the "x" component of the diffusion tensor D . The latter is connected with the CF conductivity through the Einstein relation $\sigma_{\alpha\beta}^{cf} = Ne^2 D_{\alpha\beta}$ (N is the CF density of states). Substituting Eq. (11) into this relation we obtain the expression for this diffusion correction to σ_{xx}^{cf} which coincides with the last term in Eq. (8).

According to the HLR theory, the 2DEG resistivity tensor ρ equals: $\rho = \rho^{cf} + \rho^{cs}$ where ρ^{cf} is the CF resistivity tensor ($\rho^{cf} = (\sigma^{cf})^{-1}$) and the contribution ρ^{cs} arises due to a fictitious magnetic field which originates from the Chern-Simons formulation of the theory. The latter has only off diagonal elements. Hence the diagonal components of the 2 DEG resistivity tensor coincide with the corresponding components of the CF resistivity tensor ρ^{cf} . After straightforward calculations we arrive at the result:

$$\rho_{xx} \approx \frac{1}{\sigma_0} \left\{ 1 + \left(\frac{\Delta B}{B_{eff}} \right)^2 \chi_1(gR) \right\}^{-1}; \quad (12)$$

$$\rho_{yy} \approx \frac{1}{\sigma_0} \left\{ 1 + \left(\frac{\Delta n}{n} k_F l \right)^2 \frac{\chi_2(gR)}{1 + (\Delta B / B_{eff})^2 \chi_3(gR)} \right\}. \quad (13)$$

Here $\chi_i(gR) = \alpha_i J_0^2(gR) + J_1^2(gR)$ ($i = 1, 2, 3$) and the coefficients α_i are given by the expressions:

$$\alpha_2 = 0; \quad \alpha_1 = -\frac{1}{2} \frac{(\Omega\tau)^2}{1 + (\Omega\tau)^2}; \quad \alpha_3 = \frac{(\Omega\tau)^2}{1 + (\Omega\tau)^2}. \quad (14)$$

When the density modulation is very weak $[(\Delta n/n)k_F l \ll 1]$ the corrections to the magnetoresistivity are small and we can neglect them. Under this

condition the inhomogeneity of the effective magnetic field does not significantly affect dc transport. For stronger modulation $[(\Delta n/n)k_F l \sim 1]$ the resistivity component ρ_{\perp} can be significantly changed due to the modulation. Keeping only the greatest correction (in the small parameter $\Delta B/B_{eff}$) we can write for this component of the magnetoresistivity the following approximate expression:

$$\rho_{\perp} \approx \frac{1}{\sigma_0} \left\{ 1 + \left(\frac{\Delta n}{n} k_F l \right)^2 J_1^2(gR) \right\}.$$

We also see that the resistivity component ρ_{\parallel} can be considerably changed due to the modulation:

$$\rho_{\parallel} \approx \frac{1}{\sigma_0} \left[1 + \left(\frac{\Delta n}{n} k_F l \right)^2 \frac{1}{(\Omega\tau)^2} \left(\frac{1}{2} J_0^2(gR) - J_1^2(gR) \right) \right]. \quad (15)$$

Our result for ρ_{\perp} is in general agreement with the previous theory (See e.g. Ref. [10]) For $gR \gg 1$ our formula coincides with the corresponding results of [10, 11] for a simple harmonic modulation. Apart from $\nu = 1/2$ when the parameter gR is of the order of or smaller than unity (i.e. the radius of the CF cyclotron orbit is of the order of or smaller than the modulation wavelength), we have to modify our results for the magnetoresistivity component ρ_{\perp} . The reason is that our approximation for ρ_{\perp} is valid within the relaxation time approximation in the Boltzmann equation with relaxation towards the total instead of the local equilibrium distribution function (See Ref. [10]). For $gR \sim 1$ the difference between the total and local distribution functions becomes significant. It follows from the results of a detailed and systematic analysis [10,13,14] that for $gR \sim 1$ we have to modify the correction term arising due to the modulations in the expression for ρ_{\perp} . The function $J_1^2(gR)$ has to be replaced by $J_1^2(gR)/(1 - J_0^2(gR))$. This denominator originates from the back scattering term in the collision operator in the Boltzmann equation which describes relaxation towards the local equilibrium. Thus it is the consequence of the continuity equation for CFs.

It was shown in Ref. [10] that the expression for the magnetoresistivity component ρ_{\parallel} is not changed if we assume the relaxation is towards either local or total equilibrium. Thus we can use our semiquantitative expression (15) for ρ_{\parallel} within the range of strong effective magnetic fields when $gR < 1$. We remark that our result (15) for ρ_{\parallel} disagrees with the previous theory presented in Refs. [10, 13, 14, 15]. It was concluded in these papers that magnetic modulations themselves cannot influence this component of the magnetoresistivity. This discrepancy originates from the difference in the procedure of calculation of the response functions averaged over the period of modulations. Here we first introduce the average conductivity [See Eqs. (6) and (7)] and after that we convert it to the magnetoresistivity tensor, in contrast to Refs.

[10, 13, 14, 15] where the average is taken last. It enables us to keep corrections proportional to $(\Delta\Omega/\Omega)^2 J_0^2(gR)$.

These corrections are missed when we first calculate the resistivity tensor and then average it over the period of modulation as in Ref. [10]. Another source of the discrepancy is the diffusion correction included to σ_{xx} . However this term has a clear physical sense and originates from the extra current along the "x" direction due to the guiding center drift.

For $gR < 1$ the correction to the magnetoresistivity component ρ_{\perp} takes on values of the order of unity and increases upon increase of the effective magnetic field. This corresponds to a minimum in the magnetic field dependence of ρ_{\perp} around $\nu = 1/2$. Such a minimum was observed in experiments [1, 12]. The magnetic field dependence of the resistivity component ρ_{\parallel} for $gR \sim 1$ is more complicated. Within a certain range of gR we may observe a maximum of ρ_{\parallel} about $\nu = 1/2$. The maximum can be developed when the CF mean free path and the modulation period are large enough to obey the inequalities $l > R$ and $gR < 1$ for sufficiently small B_{eff} . Within another interval of gR this maximum can be converted to a minimum and for some values of gR this component of the magnetoresistivity will exhibit only a very weak dependence on the effective magnetic field. The character of this dependence is determined by the relation between Bessel functions $J_0^2(gR)$ and $J_1^2(gR)$ included in the second term of our expression (15).

We conjecture that by changing the modulation wavelength λ we can observe all three kinds of the dependence of ρ_{\parallel} upon the magnetic field for the same experimental sample. Actually a very weak dependence of the resistivity component ρ_{\parallel} on the effective magnetic field near $\nu = 1/2$ was observed in the experiments of Ref. [12] and a maximum was observed in the experiments of Ref. [1]. In both cases the resistivity component ρ_{\perp} exhibits a large maximum about $\nu = 1/2$.

We tried to describe the maximum in the magnetic field dependence of the longitudinal magnetoresistivity about $\nu = 1/2$ using our result (15) for the parameters close to the parameters of the corresponding experiments of [1]: $n \sim 1.1 \times 10^{11} \text{ cm}^{-3}$, $p_F \sim 1.2 \times 10^{-21} \text{ g cm/s}^2$, $\lambda \sim 1.5 \mu\text{m}$, $l \sim 10^{-4} \text{ cm}$ and we obtained a maximum in the longitudinal magnetoresistivity near $\nu = 1/2$. The magnitude of this maximum strongly depends on the magnitude of the density modulations [see Fig. 1(a)]. However for a circular CF-FS the maximum about $\nu = 1/2$ is noticeably smaller in magnitude than the experimental results of [1] for reasonable values of $\Delta n/n$. We can reach much better agreement with this experiment for a noncircular CF-FS. In principle, a noncircular shape of the CF-FS may originate from the effect of the crystalline field of adjacent layers of GaAs/AlGaAs. For clean samples like those used in the experiments of [1] this effect can be noticeable. Suppose that the CF-FS is not a circle but an ellipse. In this case the parameter

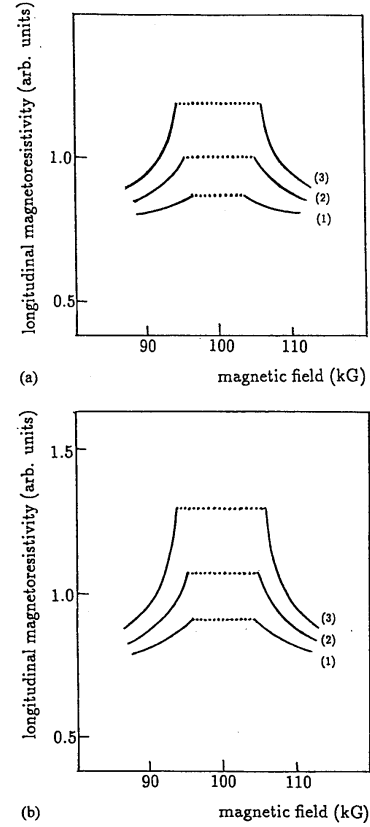


FIG. 1: The magnetic field dependence of the longitudinal dc resistivity near $\nu = 1/2$ for a circular (a) and elliptic (b) CF-FS for different magnitudes of density modulations. Curves are plotted for $l \sim 10^{-4} \text{ cm}$, $\lambda = 1.3 \mu\text{m}$, $\Delta n/n = 0.03(1)$; $0.05(2)$; $0.07(3)$. The dotted part of the theoretical curve corresponds to the region of the values of B_{eff} where Eq. (15) cannot be applied.

$gR = gp_1 c / |e| B_{eff}$ includes the maximum value of the CF quasimomentum component p_x which does not coincide with p_F . This can give us a noticeable enhancement in the magnitude of the maximum as is shown in Fig. 1(b). Plotting the curves in this figure we assumed that p_1 is $\sim 1.0 \times 10^{-21} \text{ g cm/s}^2$. Using this value of p_1 we get reasonable agreement with the experimental data of Willett et al as is shown on Fig. 2.

Our simplified formula (20) cannot be applied to the region immediately adjacent to $\nu = 1/2$ where the condition $\Delta B / B_{eff} \ll 1$ is not satisfied. So we cannot analyze the effect of channeled orbits of CFs which occur immediately adjacent to $\nu = 1/2$ where the effective magnetic field is of the same order of magnitude as the oscillating correction ΔB . To analyze the dc response of the modulated 2DEG for $\Delta B \simeq B_{eff}$ we have to solve the CF transport problem consistently for example following the way treated in Ref. [10]. Nevertheless our semiquantitative approach captures the essential physics of the considered effect and gives simple analytical results applicable

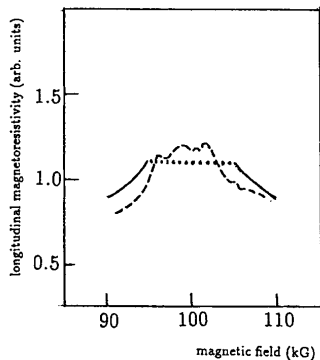


FIG. 2: dc magnetoresistivity versus the effective magnetic field. Dashed line – experiment of Ref. [1], solid line – theory for the parameters $n = 1.1 \times 10^{11} \text{ cm}^{-2}$, $\Delta n/n = 0.03$, $\lambda = 1.5 \mu\text{m}$, $l = 10^{-4} \text{ cm}$. The dotted part of the theoretical curve corresponds to the region of the values of B_{eff} where Eq. (15) cannot be applied.

for the comparison with experimental data.

In summary, we show that within the single relaxation time approximation we obtain good semiquantitative agreement with experimental observation of a maximum in parallel resistivity $\rho_{||}$, observed in the experiments of Ref. [1]. We predict that the magnetic field dependence of this component of the magnetoresistivity can be of different character for different gR i.e by changing the wavelength of modulation the maximum in $\rho_{||}$ can be converted to a minimum or to a negligibly weak dependence of this resistivity component upon the effective magnetic field. This maximum as well as the minimum in the magnetic field dependence of the resistivity ρ_{\perp} about $\nu = 1/2$ arises due to the inhomogeneity of the effective magnetic field in a modulated 2DEG near one half filling.

We thank W.R. Willett for kindly giving us a preprint of reference [1] and G.M. Zimbovsky for help with the manuscript. We also thank S. Das Sarma for discussions of possible asymmetry effects on the CF-FS due to the crystal field present on the ultra clear surfaces of Al-GaAs, Used in the magnetotransport experiments. Support from a PSC-CUNY FRAP "In – Service" Award is acknowledged.

-
- [2] R.R. Gerhardts, D. Weiss and K. von Klitzing, Phys. Rev. Lett. **62**, 1173 (1989).
 - [3] R.W. Winkler, J.P. Kotthaus and K. Ploog, Phys. Rev. Lett. **62**, 1177 (1989).
 - [4] H.A. Carmona, A.K. Geim, A. Nogaret, P.C. Main, T.J. Foster, M. Henini, S.P. Beaumont, and M.G. Blamire, Phys. Rev. Lett. **74**, 3009 (1995).
 - [5] P.D. Ye, D. Weiss, R.R. Gerhardts, M. Seeger, K. von Klitzing, K.Eberl, and H. Nickel, Phys. Rev. Lett. **74**, 3013 (1995).
 - [6] C. Zhang and R.R. Gerhardts, Phys. Rev. B **41**, 12850 (1990).
 - [7] F.M. Peeters and P. Vasilopoulos, Phys. Rev. B **47**, 1466 (1993).
 - [8] D. Pfannkuche and R.R. Gerhardts, Phys. Rev. B **46**, 12606 (1992).
 - [9] C.W.J. Beenakker, Phys. Rev. Lett. **62**, 2020 (1989).
 - [10] R. Menne and R.R. Gerhardts, Phys. Rev. B **57**, 1707 (1998).
 - [11] R.R. Gerhardts, Phys. Rev. B **53**, 11064 (1996).
 - [12] J.H. Smet, K. von Klitzing, D. Weiss and W. Wegscheider, Phys. Rev. Lett. **80**, 4538 (1998).
 - [13] A.D. Mirlin, P. Wolfle, Y. Levinson and O. Entin-Wohlman, Phys. Rev. Lett. **81**, 1070 (1998).
 - [14] F. von Oppen, A. Stern, B. Halperin, Phys. Rev. Lett. **80** 4494 (1998).
 - [15] S.D.M. Zverschke and R.R. Gerhardts, Phys. Rev. Lett. **83**, 2616 (1999).
 - [16] B.I. Halperin, P.A. Lee and N. Read, Phys. Rev. B **47**, 7312 (1993).
 - [17] B.I. Halperin, in: S. Das Sarma and A. Pinczuk (Eds.) *Low Dimensional Semiconductor Structures*, Wiley, New-York, 1996.
 - [18] D.-H. Lee, Phys. Rev. Lett. **79**, 4745 (1997).
 - [19] R. Shankar and G. Murthy, Phys. Rev. Lett. **79**, 4437, (1997).
 - [20] V. Pasquier and F.D.M. Haldane, cond-mat/9712169.
 - [21] A. Stern, B.I. Halperin, F. von Oppen, S.H. Simon, Phys. Rev. B **59**, 12547 (1999).
 - [22] N.A. Zimbovskaya and J.L. Birman, Phys. Rev. B **60**, 16762 (1999).
 - [23] An alternative theoretical description of quantum Hall system at $\nu = 1/2$ was recently proposed (see Refs. [18, 19, 20]). In the framework of this approach the relevant quasiparticles of the system are neutral dipolar fermionic objects. However a magneto-transport theory based on this new physical picture [21] gives similar results for observables.

[1] R.W. Willett, K.W. West and L.N. Pfeiffer, Phys. Rev. Lett. **83**, 2624 (1999).